t = 0 $\frac{1}{2}$

$$X(t)=\begin{cases}\cos \pi t, & t \\ 2t & t\end{cases}$$

1 X(t) $F\left(\frac{1}{2};x\right)$ F(1;x).

2 X(t) $F\left(\frac{1}{2},1;x_1,x_2\right)$.

3 X(t) $m_X(t)$ $m_X(1)$ ${\sigma_X}^2(t)$ ${\sigma_X}^2(1)$.

参考答案：

(1) . $X_{1/2}=\begin{cases}0 & \frac{1}{2} \\ 1 & \frac{1}{2}\end{cases}$, $X_1=\begin{cases}-1 & \frac{1}{2} \\ 1 & \frac{1}{2}\end{cases}$

$$F\left(\frac{1}{2};x\right)=P\{X_{1/2}\le x\}=\begin{cases}0, & x\le 0\\ \frac{1}{2}, & 0\le x<1\\ 1, & x\ge 1\end{cases}$$

$$F(1;x)=P\{X_1\le x\}=\begin{cases}0, & x\le -1\\ \frac{1}{2}, & -1\le x<2\\ 1, & x\ge 2\end{cases}$$

(2) . $X_{1/2}$,X_1

X

$$F\left(\frac{1}{2}\quad 1; x_1, x_2\right) = \begin{cases} \frac{1}{4}, & 0 \le x_1 < 1, -1 \le x_2 < 2 \\ \frac{1}{2}, & 0 \le x_1 < 1, x_2 \ge 2 \quad x_1 \ge 1 \quad -1 \le x_2 < 2 \\ 1, & x_1 \ge 1 \quad x_2 \ge 2 \end{cases}$$

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s

Θ A $\cos(\omega t+\Theta)$ A

$\cos(\omega t+\Theta)\cos(\omega s+\Theta)$

$$\mathrm{EX(t)}=\mathrm{E}[\mathrm{A}\cos(\omega t+\Theta)]=\mathrm{EA}\cdot\mathrm{E}\cos(\omega t+\Theta)=0$$

$$\begin{aligned}&\mathrm{E}\cos(\omega t+\Theta)\cos(\omega s+\Theta)\\&=\mathrm{E}\left[\frac{1}{2}\cos\omega(s-t)+\frac{1}{2}\cos(\omega s+\omega t+2\Theta)\right]\\&=\frac{1}{2}\cos\omega(s-t)+\frac{1}{4\pi}\int_0^{2\pi}\cos[\omega(t+s)+2\Theta]\,d\Theta\\&=\frac{1}{2}\cos\omega(s-t)\end{aligned}$$

$$\mathrm{E}A^2=\int_0^{\infty}\frac{a^3}{\sigma^2}\exp\left(-\frac{a^2}{2\sigma^2}\right)da=\int_0^{\infty}(-a^2)\,d\left(\exp\left\{-\frac{a^2}{2\sigma^2}\right\}\right)$$

$$=-a^2\exp\left\{-\frac{a^2}{2\sigma^2}\right\}\Big|_0^{\infty}+2\int_0^{\infty}a\exp\left\{-\frac{a^2}{2\sigma^2}\right\}da$$

$$=-2\sigma^2\exp\left\{-\frac{a^2}{2\sigma^2}\right\}\Big|_0^{\infty}=2\sigma^2$$

$$\begin{aligned}R_X(t,s)&=\mathrm{E}A^2[\cos(\omega t+\Theta)\cos(\omega s+\Theta)]\\&=\sigma^2\cos\omega(s-t)\end{aligned}$$

X(t) .

X_1 X_2 X_3 $N(\mu,\sigma^2)$, $U=\frac{1}{\sqrt{3}}(X_1+X_2+X_3)$, $V=\frac{1}{\sqrt{2}}(X_1-X_2)$, $W=\frac{1}{\sqrt{6}}(X_1+X_2-2X_3)$, (U,V,W) Σ.

参考答案：

$$\mathrm{DU}=\frac{1}{3}(DX_1+DX_2+DX_3)=\sigma^2$$

$$\mathrm{DV}=\frac{1}{2}(DX_1+DX_2)=\sigma^2$$

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$$\mathrm{DW} = \frac{1}{6}(DX_1 + DX_2 + 4DX_3) = \sigma^2$$

$$\mathrm{EU} = \frac{3\mu}{\sqrt{3}} = \sqrt{3}\mu, EV = \frac{1}{\sqrt{2}}(\mu - \mu) = 0, EW = -\frac{1}{\sqrt{6}}(\mu + \mu - 2\mu) = 0$$

$$\mathrm{cov(U,V)} = \mathrm{E(UV)} - \mathrm{EU} \cdot \mathrm{EV} = \frac{1}{\sqrt{6}}(EX_1{}^2 + EX_1 \cdot EX_2 + EX_1 \cdot EX_3 - EX_1 \cdot EX_2$$
$$-\mathrm{E}X_2{}^2 - EX_2 \cdot EX_3) = 0 = \mathrm{cov}(V,U)$$

$$\mathrm{cov(U,W)} = \mathrm{E(UW)} - \mathrm{EU} \cdot \mathrm{EW}$$
$$= -\frac{1}{\sqrt{18}}\left(EX_1{}^2 + 2EX_1 \cdot EX_2 + EX_1 \cdot EX_3 + \mathrm{E}X_2{}^2 - EX_2 \cdot EX_3 - 2EX_1 \cdot EX_3\right.$$
$$\left. - 2\mathrm{E}X_3{}^2\right) = 0 = \mathrm{cov}(W,U)$$

$$\mathrm{cov(V,W)} = \mathrm{E(VW)} - \mathrm{EV} \cdot \mathrm{EW}$$
$$= -\frac{1}{\sqrt{12}}\left(EX_1{}^2 - 2EX_1 \cdot EX_3 - \mathrm{E}X_2{}^2 + 2EX_2 \cdot EX_3\right)$$
$$= 0 = \mathrm{cov(W,V)}$$

$$\Sigma = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{bmatrix}$$

$\{X(t), t \geq 0\}$ $\{X(t)\}$

参考答案： $\{X(t), t \geq 0\}$

1 $X(t) = 0$

2 $\{X(t), t \geq 0\}$

3 $\mathrm{t} > 0, X(t)$ $\mathrm{N}(0, \sigma^2 t)$

$\{X(t), t \geq 0\}$

$$\mathrm{EX(t)} = 0, \qquad t \geq 0$$

$$\gamma(\mathrm{s,t}) = \mathrm{E}[X(s)X(t)] = E\{X(s)[X(t) - X(s) + X(s)]\}$$
$$= E\{X(s)[X(t) - X(s)]\} + E\{X^2(s)\}$$
$$= E[X(s)]E[X(t) - X(s)] + E[X^2(s)] \qquad (\qquad)$$
$$= \sigma^2 s = \sigma^2 \min\{s,t\}$$

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s t

$X(t) = R \cdot t + C, t \in (0, \infty)$ C R [0,1]

(1) X(t)

(2) X(t)

参考答案：

[1] R [0,1] C X(t) R

X(t) $[C, C + t]$

$$f(x) = \begin{cases} \frac{1}{t}, C \le x \le C + t \\ 0, \end{cases}$$

$$F(X) = \begin{cases} 0, x < C \\ \frac{x - C}{t}, C \le x \le C + t \\ 1, x > C + t \end{cases}$$

[2]

$$m_x(t) = E[x] = \int_C^{C+t} x \cdot \frac{1}{t} dx = \frac{t}{2} + C$$

$$R_X(s, t) = E[X(s)X(t)] = \frac{1}{3}st + \frac{C}{2}(s + t) + C^2$$

$$\gamma_X(s, t) = E\left[\left(X(s) - m_x(s)\right)\left(X(t) - m_x(t)\right)\right] = \frac{st}{12}$$

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$X(n) = sin(Un), n \in N$ U 0, 2π

$X(n)$

参考答案：

$$EX(n) = \int_0^{2\pi} \sin(tn)\frac{1}{2\pi}dt = 0$$

$$R_X(n, n+\tau) = E\{X(n)X(n+\tau) = \int_0^{2\pi} \sin(tn)\sin(t(n+\tau))\frac{1}{2\pi}dt$$

$$= \int_0^{2\pi} \cos(t\tau) + \cos(t(2n+\tau))\frac{1}{2\pi}dt$$

$$= \int_0^{2\pi} \cos(t\tau)\frac{1}{2\pi}dt = \sin(2\pi\tau)/2\pi\tau = 0$$

τ

$\{X(t), t \in T\}$ $\mu_x(t)$ $\gamma_x(t_1, t_2)$ $\varphi(t)$

$\{Y(t) = X(t) + \varphi(t)\}$

参考答案： $\{Y(t)\}$:

$E[Y(t)] = E[X(t) + \varphi(t)] = \mu_X(t) + \varphi(t)$

$\{Y(t)\}$:

$\gamma_Y(s, t)$

$= E\big[Y(s)Y(t) - E\big(Y(s)\big)E\big(Y(t)\big)\big]$

$= E\big[\big(X(s) + \varphi(s)\big)\big(X(t) + \varphi(t)\big) - \big(\mu_X(s) + \varphi(s)\big)\big(\mu_X(t) + \varphi(t)\big)\big]$

$= E[X(s)X(t) - \mu_X(s)\mu_X(t)] = \gamma_X(s, t)$

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